HANDOUT 8: VARIABLES AND QUANTIFICATION

From now on, we are assuming that the inputs for semantic interpretation are Logical Form representations like the ones given below. A Logical Form for a quantified sentence has three parts: Quantifier, (domain) restriction, and nuclear scope. We are now in a position to specify what the semantic contribution of each of those parts is. A quantified sentence denotes a set of possible situations in which a certain relation holds between a set A and a set B. The set A is contributed by the restriction of the quantifier. The set B is provided by the nuclear scope. We are already familiar with the different relations provided by different quantifiers. And we have been familiar with the interpretation of common nouns for quite some time. What we have just learned is how to interpret open sentences. This means that we are now ready to build up the denotations of simple Logical Forms from its three parts. Let's look at three examples to help us figure out how to design a compositional semantic interpretation procedure.

Case 1

[No woman]$_x$ x adores x

Intended interpretation

For any variable assignments g:

{s: [[**woman**]]^g(s) ∩ {b: s ∈ [[**x adores x**]]$^{x \rightarrow b}$ } = ∅ }

At first glance, the intended interpretation above may not seem very intuitive, but looking at it piece by piece will reveal how everything works. The quantifier **no** gives us A ∩ B = ∅, which tells us that no member of A is a member of B. The sets A and B are determined by the domain restriction **woman** and nuclear scope **x adores x**, respectively. We know that the sentence should mean no woman adores herself, and so we know what sets we want. From **woman** we would like the set of individuals that are women in s, and from **x adores x** we would like the set of individuals who adore themselves in s. Together with **no**, that would give us:

{a: a is a woman in s} ∩ {b: b adores b in s} = ∅

The above is the condition we want to impose on the set of situations delivered by the whole sentence:

{s: {a: a is a woman in s} ∩ {b: b adores b in s} = ∅}

Getting the set of women in s from **woman** is straightforward. We just need to take its denotation and apply it to the situation: [[**woman**]]^{g}(s). The only tricky part , then, is getting the set of individuals that adore themselves from **x adores x**, the open sentence that is our nuclear scope. The denotation of a sentence (open or not) is a set of possible situations, not a set of individuals. The following partial computation shows how we can get from a set of possible situations to a set of individuals. The key is in the variable assignment.

{s: [[**woman**]]^{g}(s) ∩ {b: s ∈ [[**x adores x**]]$^{x \rightarrow b}$ } = ∅ }

= {s: {a: a is a woman in s} ∩ {b: s ∈ {s: b adores b in s}} = ∅ }   Earlier computation[1].
= {s: {a: a is a woman in s} ∩ {b: b adores b in s} = ∅ }   Set Theory

The key here is in the variable assignment. We can paraphrase {b: s ∈ [[**x adores x**]]$^{x \rightarrow b}$ } as the set of all b such that the situation s is a member of the set of all situations such that b adores b in those situations. In other words, the set of all b such that b adores b in s. The variable assignment **x** → b makes sure that the we are getting the set of all b that play that particular role in the situation.

[1] Remember: {a: a adores a in s} = {b: b adores b in s} = {c: c adores c in s} = ...etc.

Case 2

[Every woman]$_x$ x adores x

Intended interpretation

For all variable assignments g:

{s: [[**woman**]]^g(s) ⊆ {b: s ∈ [[**x adores x**]]$^{x \to b}$ } }

= {s: {a: a is a woman in s} ⊆ {b: s ∈ {s: b adores b in s} } } Previous comp.

= {s: {a: a is a woman in s} ⊆ {b: b adores b in s} } Set Theory

Case 3

[Some woman]$_x$ x adores x

Intended interpretation

For all variable assignments g:

{s: [[**woman**]]^g(s) ∩ {b: s ∈ [[**x adores x**]]$^{x \to b}$ } ≠ ∅ }

= {s: {a: a is a woman in s} ∩ {b: s ∈ {s: b adores b in s} } ≠ ∅ } Previous comp.

= {s: {a: a is a woman in s} ∩ {b: b adores b in s} ≠ ∅ } Set Theory

Now we can work on putting things together compositionally.

The compositional interpretation of Logical Forms

Additions to the lexicon

For all variable assignments g and all possible situations s, we have:
[[**woman**]]^g(s) = {a: a is a woman in s}, for all variable assignments g.

Etc.

Additions to the semantic interpretation rules

For all variable assignments g we have:

S3 If α is a common noun and β is a sentence, then [**no** $\alpha]_{\mathbf{x}}\, \beta$ is a sentence, and

$$[\![\ [\mathbf{no}\ \alpha]_{\mathbf{x}}\, \beta\ ]\!]^{g} = \{s: [\![\ \alpha\ ]\!]^{g}(s) \cap \{b: s \in [\![\ \beta\ ]\!]^{\mathbf{x} \to b}\} = \varnothing\ \}$$

S4 If α is a common noun and β is a sentence, then [**every** $\alpha]_{\mathbf{x}}\, \beta$ is a sentence, and

$$[\![\ [\mathbf{every}\ \alpha]_{\mathbf{x}}\, \beta\ ]\!]^{g} = \{s: [\![\alpha]\!]^{g}(s) \subseteq \{b: s \in [\![\ \beta\ ]\!]^{\mathbf{x} \to b}\}\ \}$$

S5 If α is a common noun and β is a sentence, then [**some** $\alpha]_{\mathbf{x}}\, \beta$ is a sentence, and

$$[\![\ [\mathbf{some}\ \alpha]_{\mathbf{x}}\, \beta\ ]\!]^{g} = \{s: [\![\ \alpha\ ]\!]^{g}(s) \cap \{b: s \in [\![\ \beta\ ]\!]^{\mathbf{x} \to b}\} \neq \varnothing\ \}$$

Putting variable assignments to work

With what we've learned about variable assignments, along with the new additions to our lexicon and rules, we can now compositionally interpret the logical forms of sentences like that in (1), and we will do so. The section after this example contains a summary of all of the rules that we have developed so far. You should understand how they work together knowing, however, that we will be making a few more changes before we are done.

(1) **No woman adores herself**.

The Logical Form of sentence (1):

[No	woman]x	x adores x.
Q	**restriction**	**nuclear scope**

For any variable assignment g, $[[\ [\textbf{No woman}]_x\ \textbf{x adores x}\ ]]^g =$

1. $\{s: [[\textbf{woman}]]^g(s) \cap \{b: s \in [[\textbf{x adores x}\ ]]^{x \to b}\} = \varnothing\}$ S3
2. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: s \in [[\textbf{x adores x}\ ]]^{x \to b}\} = \varnothing\}$ Lex.
3. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: s \in \{s: [[\textbf{x}]]^{x \to b} \in [[\textbf{adores x}]]^{x \to b}(s)\}\} = \varnothing\}$ S1
4. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: [[\textbf{x}]]^{x \to b} \in [[\textbf{adores x}]]^{x \to b}(s)\} = \varnothing\}$ CP
5. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: b \in [[\textbf{adores x}]]^{x \to b}(s)\} = \varnothing\}$ Lex.
6. $= s: \{a: a \text{ is a woman in } s\} \cap \{b: b \in \{a: \langle a,[[\textbf{x}]]^{x \to b}\rangle \in [[\textbf{adores}]]^{x \to b}(s)\}\}= \varnothing\}$ S2
7. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: \langle b,[[\textbf{x}]]^{x \to b}\rangle \in [[\textbf{adores}]]^{x \to b}(s)\}= \varnothing\}$ CP
8. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: \langle b,b\rangle \in [[\textbf{adores}]]^{x \to b}(s)\}= \varnothing\}$ Lex.
9. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: \langle b,b\rangle \in \{\langle a,b\rangle: a \text{ adores } b \text{ in } s\}\}= \varnothing\}$ Lex.
10. $= \{s : \{a: a \text{ is a woman in } s\} \cap \{b: b \text{ adores } b \text{ in } s\}= \varnothing\}$ CP

The above is the set of possible situation s such that the intersection of individuals that are women in s and individuals that adore themselves in s is empty. Thus, it is the set of possible situations in which no woman adores herself.

Note: In the above, Lex stands for Lexicon and CP stands for Comprehension Principle

Summary of rules we are working with (at the moment)

For all variable assignments g and all possible situations s, we have:

Names

[[**Ann**]]^g = Ann. [[**Mary**]]^g = Mary, etc.

Variables

[[**x**]] ^{g} = g(**x**).

Common nouns

[[**woman**]]^g(s) = {a: a is a woman in s}, etc.

Intransitive verbs

[[**slept**]]^g(s) = {a: a slept in s}, etc.

Transitive verbs

[[**adores**]]^g(s) = {<a,b>: a adores b in s}, etc.

S1 If α is a name, and β is an intransitive verb or a VP, then αβ is a sentence, and [[αβ]]^g = {s: [[α]]^g ∈ [[β]]^g(s) }.

S2 If α is a name and β is a transitive verb, then βα is a VP, and for all possible situations s, [[βα]]^g(s) = {a: <a,[[α]]^g> ∈ [[β]]^g(s) }.

S3 If α is a common noun and β is a sentence, then [**no** α]$_{\mathbf{x}}$ β is a sentence, and [[[**no** α]$_{\mathbf{x}}$ β]]^g = {s: [[α]]^g(s) ∩ {b: s ∈ [[β]]$^{\mathbf{x}\to b}$ } = ∅ }

S4 If α is a common noun and β is a sentence, then [**every** α]$_{\mathbf{x}}$ β is a sentence, and [[[**every** α]$_{\mathbf{x}}$ β]]^g = {s: [[α]]^g(s) ⊆ {b: s ∈ [[β]]$^{\mathbf{x}\to b}$ } }

S5 If α is a common noun and β is a sentence, then [**some** α]$_{\mathbf{x}}$ β is a sentence, and [[[**some** α]$_{\mathbf{x}}$ β]]^g = {s: [[α]]^g(s) ∩ {b: s ∈ [[β]]$^{\mathbf{x}\to b}$ } ≠ ∅ }

Multiple Quantifiers

We still have some work to before we can compositionally interpret structures with multiple quantifiers.

Examples of relevant constructions

1. Some woman saw every movie.
   [Some woman]$_x$ [every movie]$_y$ x saw y

2. Some woman saw every movie.
   [Every movie]$_y$ [Some woman]$_x$ x saw y

Variable assignments. The general case

We have seen that one way of guaranteeing that both open and closed sentences receive an appropriate denotation is to let the semantic interpretation function [[ ]] depend on a variable assignment. Consequently, all sentences and their parts will be interpreted relative to a variable assignment g. When we encountered variable assignments for the first time, our Logical Form representations contained only one variable, the variable **x**. This made it possible to think of variable assignments as pairings of **x** with an individual. The Logical Form representations that we will be looking at in this section involve more then one variable. We therefore need to introduce a more general notion of variable assignment.

Definition of variable assignments for a language L

Let E be the set of individuals (the universe of discourse). A variable assignment for a language L pairs every variable of L with exactly one member of E. A variable assignment, then, is a function from the set of variables of L to E.

Notational Convention

For any individual a ∈ E, any variable u, and any variable assignment g, $g^{u \to a}$ is that variable assignment that is just like g except (possibly) that g assigns a to u.

Example:

Assume that L has only three variables **x**, **y**, and **z**. Consider now the following variable assignment g:

g: **x** → Jan
y → Joshua
z → Joshua

The variable assignment $g^{y \to Ann}$ is the following function:

$$g^{y \to Ann}: \begin{bmatrix} \mathbf{x} \to \text{Jan} \\ \mathbf{y} \to \text{Ann} \\ \mathbf{z} \to \text{Joshua} \end{bmatrix}$$

The notation introduced above can be iterated.

We may consider $(g^{y \to Ann})^{z \to Maria}$, for example. We have then:

$$(g^{y \to Ann})^{z \to Maria}: \begin{bmatrix} \mathbf{x} \to \text{Jan} \\ \mathbf{y} \to \text{Ann} \\ \mathbf{z} \to \text{Maria} \end{bmatrix}$$

Our last fragment

Remember that in what follows, a variable assignment is any function that assigns to every variable of the language an element of the universe of discourse E.

For any variable assignment g and any possible situation s we have:

The lexicon

Names (proper names)

[[**Ann**]]^g = Ann [[**Mary**]]^g = Mary

Names (variables)

[[**x**]]^g = g(**x**) [[**y**]]^g = g(**y**) [[**z**]]^g = g(**z**)

Intransitive verbs

[[**slept**]]^g(s) = {a: a slept in s}.

Transitive verbs

[[**saw**]]^g(s) = {<a,b>: a saw b in s}.

Common nouns

[[**woman**]]^g(s) = {a: a is a woman in s}.

[[**movie**]]^g(s) = {a: a is a movie in s}.

The semantic interpretation rules

For all variable assignments g and possible situations s we have:

S1 If α is a name, and β is an intransitive verb or a VP, then αβ is a sentence, and [[αβ]]^g = {s: [[α]]^g ∈ [[β]]^g(s) }.

S2 If α is a name and β is a transitive verb, then βα is a VP, and [[βα]]^g(s) = {a: <a,[[α]]^g> ∈ [[β]]^g(s) }.

S3 If α is a common noun, β is a sentence, and u is a variable, then [**no** α]$_u$ β is a sentence, and [[[**no** α]$_u$ β]]^g = {s: [[α]]^g(s) ∩ {c: s ∈ [[β]]$^{g\ u\rightarrow c}$ } = ∅ }.

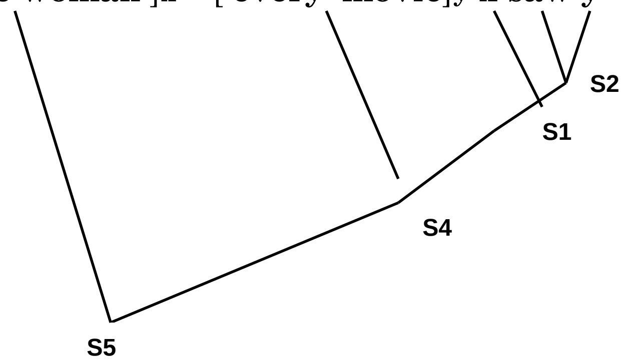
S4 If α is a common noun, β is a sentence, and u is a variable, then [**every** α]$_u$ β is a sentence, and [[[**every** α]$_u$ β]]^g = {s: [[α]]^g(s) ⊆ {c: s ∈ [[β]]$^{g\ u\rightarrow c}$ } }.

S5 If α is a common noun, β is a sentence, and u is a variable, then $[\textbf{some}\ \alpha]_u\ \beta$ is a sentence, and $[[\ [\textbf{some}\ \alpha]_u\ \beta]]^g = \{s:\ [[\alpha]]^g(s) \cap \{c: s \in\ [[\beta]]^{g\ u \rightarrow c}\} \neq \emptyset\ \}$.

Note: When applying those rules in an actual computation, you may have to rename the variables used to specify sets so as to avoid variable confusion.

Computation of the denotation of Logical Form (1)

[Some woman]$_x$ [every movie]$_y$ x saw y



For all variable assignments g we have:

1. $[[\ [\textbf{Some woman}]_x\ [\textbf{every movie}]_y\ \textbf{x saw y}\ ]]^g =$

   $= \{s: [[\ \textbf{woman}\ ]]^g(s) \cap \{\ c: s \in [[\ [\ \textbf{every movie}\ ]_y\ \textbf{x saw y}\ ]]^{g\ x \rightarrow c}\ \} \neq \emptyset\ \}$.

   by S5.

2. $\{s: [[\ \textbf{woman}\ ]]^g(s) \cap \{\ c: s \in \{s' : [[\ \textbf{movie}\ ]]^{g\ x \rightarrow c}(s') \subseteq$
   $\{d: s' \in [[\ \textbf{x saw y}\ ]]^{(g\ x \rightarrow c)y \rightarrow d}\ \}\ \}\ \} \neq \emptyset\ \}$

   From (1), by S4, but with renaming meta-language variables so as to avoid variable confusion.

3. $\{s: \{a: \text{a is a woman in s}\ \} \cap \{\ c: s \in \{s' : \{a: \text{a is a movie in s'}\}\ ) \subseteq$
   $\{d: s' \in [[\ \textbf{x saw y}\ ]]^{(g\ x \rightarrow c)y \rightarrow d}\ \}\ \}\ \} \neq \emptyset\ \}$

   From (2), lexicon, 2 times.

4. {s: {a: a is a woman in s } ∩ { c: {a: a is a movie in s} ⊆
{d: s ∈ [[**x saw y**]]$^{(g\ x\to c)y\to d}$ } } ≠ ∅ }
From (3), by set theory.

5. {s: {a: a is a woman in s } ∩{ c: {a: a is a movie in s} ⊆
{d: s ∈{s': [[**x**]]$^{(g\ x\to c)y\to d}$ ∈ [[**saw y**]]$^{(g\ x\to c)y\to d}$(s') } } } ≠ ∅ }
From (4), by S1.

6. {s: {a: a is a woman in s } ∩{ c: {a: a is a movie in s} ⊆
{d: [[**x**]]$^{(g\ x\to c)y\to d}$ ∈ [[**saw y**]]$^{(g\ x\to c)y\to d}$(s) } } ≠ ∅ }
From (5), by set theory.

7. {s: {a: a is a woman in s } ∩ { c: {a: a is a movie in s} ⊆
{d: c ∈ [[**saw y**]]$^{(g\ x\to c)y\to d}$(s) } } ≠ ∅ }
From (6), lexicon.

8. {s: {a: a is a woman in s } ∩ { c: {a: a is a movie in s} ⊆
{d: c ∈ {a: <a,[[**y**]]$^{(g\ x\to c)y\to d}$> ∈ [[**saw**]]$^{(g\ x\to c)y\to d}$(s) } } } ≠ ∅ }
From (7), S2.

9. {s: {a: a is a woman in s } ∩{ c: {a: a is a movie in s} ⊆
{d: c ∈ {a: <a, d> ∈ [[**saw**]]$^{(g\ x\to c)y\to d}$(s) } } } ≠ ∅ }
From (8), lexicon.

10. {s: {a: a is a woman in s } ∩ { c: {a: a is a movie in s} ⊆
{d: <c, d> ∈ [[**saw**]]$^{(g\ x\to c)y\to d}$(s) } } ≠ ∅ }
From (9), by set theory.

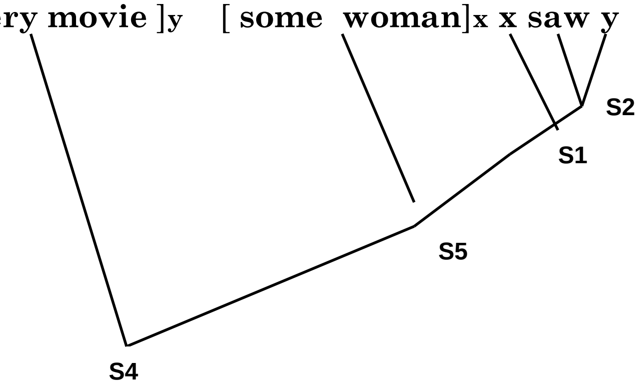
11. {s: {a: a is a woman in s } ∩ { c: {a: a is a movie in s} ⊆
{d: <c, d> ∈ {<a,b>: a saw b in s } } } ≠ ∅ }
From (10), lexicon.

12. $\{s: \{a: a \text{ is a woman in } s\} \cap \{c: \{a: a \text{ is a movie in } s\} \subseteq$
$\{d: c \text{ saw } d \text{ in } s\}\} \neq \emptyset\}$
From (11) by set theory.

That (12) specifies the correct denotation for our sentence can be seen from the following correct paraphrase: There is at least one woman in the set of individuals c such that all the movies are a subset of the things c saw.

Now let's do the computation for the denotation of Logical form (2).

[**Every movie** $]_y$ [**some woman**$]_x$ **x saw y**



For all variable assignments g we have:

1. $[[\ [\textbf{Every movie}]_y\ [\textbf{some woman}]_x\ \textbf{x saw y}\ ]]^g =$
$= \{s: [[\textbf{movie}]]^g(s) \subseteq \{c: s \in [[\ [\textbf{some movie}]_y\ \textbf{x saw y}\ ]]^{g\ y \to c}\}\}$
S4.

2. $\{s: [[\textbf{movie}]]^g(s) \subseteq \{c: s \in \{s' : [[\textbf{woman}]]^{g\ y \to c}(s') \cap$
$\{d: s' \in [[\textbf{x saw y}]]^{(g\ y \to c)x \to d}\} \neq \emptyset\}\}\}$

From (1), by S5, but with renaming meta-language variables so as to avoid variable confusion.

3. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: s \in \{s' : \{a: \text{a is a woman in } s'\} \cap$
$\{d: s' \in [[\, \textbf{x saw y} \,]]^{(g\, y \to c)x \to d} \} \neq \varnothing \} \} \}$

From (2), lexicon, 2 times.

4. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: s \in [[\, \textbf{x saw y} \,]]^{(g\, y \to c)x \to d} \} \neq \varnothing \} \}$
From (3), by set theory.

5. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: s \in \{s': [[\textbf{x}]]^{(g\, y \to c)\textbf{x} \to d} \in [[\textbf{saw y}\,]]^{(g\, y \to c)\textbf{x} \to d}(s') \} \} \neq \varnothing \} \}$
From (4), by S1.

6. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: [[\textbf{x}]]^{(g\, y \to c)\textbf{x} \to d} \in [[\textbf{saw y}\,]]^{(g\, y \to c)\textbf{x} \to d}(s) \} \neq \varnothing \}\}$

From (5), by set theory.

7. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: d \in [[\textbf{saw y}\,]]^{(g\, y \to c)\textbf{x} \to d}(s) \} \neq \varnothing \} \}$

From (6), lexicon.

8. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: d \in \{a: \langle a, [[\textbf{y}]]^{(g\, y \to c)\textbf{x} \to d} \rangle \in [[\textbf{saw}]]^{(g\, y \to c)\textbf{x} \to d}(s) \} \} \neq \varnothing \} \}$
From (7), S2.

9. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: d \in \{a: \langle a, c \rangle \in [[\textbf{saw}]]^{(g\, y \to c)\textbf{x} \to d}(s) \} \} \neq \varnothing \} \}$
From (8), lexicon.

10. $\{s: \{a: \text{a is a movie in } s\} \subseteq \{ c: \{a: \text{a is a woman in } s\} \cap$
$\{d: \langle d, c \rangle \in [[\textbf{saw}]]^{(g\, y \to c)\textbf{x} \to d}(s) \} \neq \varnothing \} \}$
From (9), by set theory.

11. $\{s: \{a: a \text{ is a movie in } s\} \subseteq \{ c: \{a: a \text{ is a woman in } s\} \cap \{d: \langle d, c \rangle \in \{\langle a,b \rangle: a \text{ saw } b \text{ in } s \} \} \neq \varnothing \} \}$

From (10), lexicon.

12. $\{s: \{a: a \text{ is a movie in } s \} \subseteq \{ c: \{a: a \text{ is a woman in } s\} \cap \{ d: d \text{ saw } c \text{ in } s \} \neq \varnothing \} \}$

From (11) by set theory.

That (12) specifies the correct denotation for our sentence can be seen from the following correct paraphrase: The set of movies is a subset of the set of individuals c such that there is at least one woman who saw c.